Fall 12-1-2012

Mathematics Curriculum Revised to Align to Common Core State Standards

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by

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December, 2012

A thesis project submitted to the

Department of Education and Human Development of the

State University of New York College at Brockport

In partial fulfillment of the requirements for the degree of

Master of Science in Education

Abstract

The Common Core State Standards (CCSS) are the first set of national standards issued in the United States. Currently, all states are in the transition from implementing individual state standards to CCSS in ELA and mathematics. As the CCSS are being introduced and implemented into classrooms, educators are struggling to update or modify their current unit, based on the National Council of Teachers of Mathematics (NCTM) Standards, to align with the new national standards. This research provides an explanation of the changes that will occur due to the CCSS in a seventh grade probability and statistics unit. An updated unit plan that has been reviewed by veteran teachers is provided. This curriculum project can be used as an instructional tool for seventh grade mathematics teachers. The focus of these lessons is on applied learning experiences, and asks students to make inferences and conjectures, which is a new requirement of the CCSS.

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Chapter 1

Introduction

The face of education is constantly changing. As technology continues to develop and replace many tasks in our daily lives, we must also consider the type of knowledge that our students require to succeed in the future. Content that was important for students to know fifty years ago has become irrelevant in today's society. Old state standards, assessment, curricula, and instructional practices are not enough to help students achieve and succeed in today's society. As educators, a huge problem that we face is receiving a group of students in our classrooms each year that have not mastered the curriculum we expected them to from the previous grade. This is an even larger problem in colleges and universities. Students arrive at two and four years colleges and universities lacking the necessary reading, writing, and mathematics knowledge and skills needed to succeed in freshman level courses. "There is abundant evidence that many students in the United States are not adequately prepared for their first college level mathematics course (Harwell et al., 2009). Most often single variable calculus is the first college level mathematics course that counts toward degree credits (Smith, 1998). There is agreement among college mathematics professors and secondary mathematics teachers that rigorous instruction promotes mathematical understanding, but there is less agreement on how to implement instruction in order to better prepare secondary students for college calculus (Harwell et at., 2009)" (Wade, 2011).

It seems that our first step to changing the face of education for the better is to change how we educate our nation's students, starting from the bottom up. We must develop better lessons that focus on useful curriculum and deeper learning, as well as standardized assessments that are consistent among the states. We must move away from memorization, drill and kill, and teaching to test; moving towards applied learning, developing a complex array of skills, and a readiness for society. Students of today should be able to graduate high school prepared to take on the opportunities ahead of them. There is no reason that every high school graduate is not "Career or college ready." The Common Core State Standards (CCSS) are here to provide a set of standards, linking all areas of the United States, to help create a set of common standards and learning goals that every high school graduate is expected to achieve, starting from kindergarten.

Problem Statement

"The CCSSI is a historic effort designed to advance nationwide education reform. Coordinated by the NGA Center and CCSSO, 48 states, the District of Columbia, Puerto Rico, and the Virgin Islands have voluntarily come together to create shared common core standards in English language arts (ELA) and mathematics. The ultimate goal is for all American children to graduate from high school ready for college, career pathways, and success in a global economy ((National Governors Association Center for Best Practices, 2010)."

. "The *Standards* do accommodate and prepare students from Algebra 1 in 8th grade, by including the prerequisites for this course in grades K-7. Students who master the K-7 material will be able to take Algebra 1 in 8th grade" (National Governors Association Center for Best Practices, 2010). Most of the changes to the old standards from New York State have occurred in the middle grades, seventh grade in particular, as this is a branching point for students to continue on to more rigorous Pre-Algebra or Algebra 1 in grade 8 (National Governors Association Center for Best Practices, 2010). Before the implementation of the CCSS, the probability and statistics unit focused on types of graphs, reading and drawing graphs, central tendency, and theoretical and experimental probability in the seventh grade. Now with the new standards, concepts such as mean, median, and mode are introduced in the sixth grade instead of seventh. The CCSS have shifted many topics previously covered at one grade level into different grade levels. This paradigm shift may cause teachers at any grade level to have to perform an overhaul on many of their current unit plans. The goal of this research is to develop a unit plan for probability and statistics for the seventh grade with engaging lessons that will cover all the necessary content aligned to the Common Core State Standards?

Significance of the Problem

Beginning in the 2012-2013 school year, students in grades 3-8 will complete end of the year assessments that are aligned to the CCSS. This means that this school year, all mathematics teachers in grades 3-8 will be required to present their units with lessons aligned to the new standards. Even though the assessments have not even been designed yet, lesson planning must begin now. The problem with this is that a significant translation of the abstract new standards must be made in order to form new concrete unit and lesson plans

Purpose

To ensure that this curriculum project aligns with the CCSS, I will ask four veteran teachers to review the lessons provided in this project. The veteran teachers will be provided I with a questionnaire to complete, which will provide multiple opinions on the same aspects of the lessons. Both criticisms and critiques will be accepted and used to improve the unit.

The goal is to research lesson ideas, activities, and teaching practices for probability and statistics practices in order to implement seventh grade mathematics lessons that are aligned to the Common Core State Standards. It is in the intent to create an innovative and engaging unit that students with foundational mathematical knowledge may use to deepen their statistical understanding and keep up with today's technologically growing society. This unit will include the tried and true elements of mathematics education with a technologically friendly unit.

Rationale

The goal is to develop a completed and well-structured seventh grade mathematics unit on probability and statistics that is aligned to the CCSS. In this way teachers will have access to this unit before the content is implemented in their seventh grade classroom.

Chapter 2

Literature Review

History of the Common Core State Standards

As a nation, we are making huge strides towards a countrywide curriculum reform in all major content areas. "Unlike past standards setting efforts, the Common Core State Standards are based on best practices in national and international education, as well as research and input from numerous sources including scholars, assessment developers, professional organizations, and educators representing all grade levels from kindergarten through postsecondary instruction. Contributions from the states and the public also served to inform the standards development process (National Governors Association Center for Best Practices, 2010)." The Common Core Standards is an initiative between the National Governors Association Center for Best Practices (NGA) and the Council of Chief State School Officers (CCSSO) that has been adopted by [45 states and 3 territories] at the time of this writing (Loertscher & Marcoux, 2010)." The development of the Common Core Standards began in 2009 in an effort to hold our students to high educational expectations in order to keep up with the demands of a rapidly changing world. Completed in 2010, "These [standards] were built with the understanding that they would be overarching and unify the expectations of student learning across the United States," replacing each states individual set of standards (Loertscher & Marcoux, 2010). Every state has agreed that they will not lower their standards from their originals; in no way is any state "dumbing-down" their curriculum to meet the Common core standards.

Currently, the NGA has developed the new common core standards for mathematics and English language arts. Their mission, "[To] provide a consistent, clear understanding of what students are expected to learn, so teachers and parents know what they need to do to help them.

The standards are designed to be robust and relevant to the real world, reflecting the knowledge and skills that our young people need for success in college and careers. With American students fully prepared for the future, our communities will be best positioned to compete successfully in the global economy" (National Governors Association Center for Best Practices, 2010).

> "The 'college and career ready' line has been based on evidence from a number of sources, including international benchmarking, surveys of postsecondary faculty and employers, review of state standards, and expert opinion. Students meeting these standards should be well-prepared for introductory mathematics courses in 2- and 4- year colleges. Still, there are persuasive reasons for students to continue on to take a fourth mathematics course in high school. Research consistently finds that taking mathematics above the Algebra II level highly corresponds to many measures of student success. In his groundbreaking report Answers in the Toolbox, Clifford Adelman found that the strongest predictor of postsecondary success is the highest level of mathematics completed (Executive Summary). ACT has found that taking more mathematics courses correlates with greater success on their college entrance examination. Of students taking (Algebra I, Geometry and Algebra II and no other mathematics courses), only thirteen percent of those students met the benchmark for readiness for college algebra. One additional mathematics course greatly increased the likelihood that a student would reach that benchmark, and three-fourths of students taking Calculus met the benchmark (ACTb 13)." (National Governors Associations Center for Best Practices, 2010).

What is different?

Steen states, "There are many ways to organize curricula. The challenge, now rarely met, is to avoid those that distort mathematics and turn off students" (2007). The CCSS have a strong focus on fewer standards than those of individual states, but instead of using broader more general statements, they aim for clarity and specificity (National Governors Association Center for Best Practices, 2010).

The standards push back all mathematics content beginning in kindergarten to help build a stronger foundation than previous standards. CCSS put extra stress on procedural skill as well as conceptual understanding; exactly what students need to ensure success as higher levels of mathematics. With these standards, "Students who have completed 7th grade and mastered the content and skills through the 7th grade will be well-prepared for algebra in grade 8. The middle school standards are robust and provide a coherent and rich preparation for high school mathematics" (National Governors Association Center for Best Practices, 2010).

The CCSS for each grade level consist of standards, which define what the students should be able to do, clusters, which summarize groups of related standards, and domains, which are larger groups of related standards (National Governors Association Center for Best Practices, 2010). But, in the actual standards, there is no explanation of an amount of time to be spent of each cluster, or which standards are most important to focus on for deep understanding. Engageny.org provides a description of the mathematics content for each grade at the "cluster level". These are provided to direct the focus and emphasis of instruction towards what it most important in the standards. Topics in each cluster do not all hold the same weight, so teachers may use these charts as a guideline for designing instruction. Table 1 is provided for the seventh grade mathematics content from Engageny.org. The bottom portion, "depth opportunities," lists the CCSS that are most important to focus on for in-depth instruction in seventh grade.

Table 1

Assignment of Weight/Importance of Topics in Seventh Grade Mathematics

Depth Opportunities Provide Standards to be Delivered with Emphasis

Grade 7		
Major	**Supporting**	**Additional**
Ratios and Proportional Relationships Analyze proportional relationships and use them to solve real-world and mathematical problems. **The Number System** Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers. **Expressions and Equations** Use properties of operations to generate equivalent expressions. Solve real-life and mathematical problems using numerical and algebraic expressions and equations.	**Statistics and Probability** Use random sampling to draw inferences about a population. 8 Investigate chance processes and develop, use, and evaluate probability models. 9	**Statistics and Probability** **o** Draw informal comparative inferences about two populations. **Geometry** o Solve real-life and mathematical problems involving angle measure, area, surface area, and volume. o Draw, construct and describe geometrical figures and describe the relationships between them.
Depth Opportunities: RP 2; NS 3; EE 3, 4; G 6		

Explanations of terms used:

Major clusters – areas of intensive focus, where students need fluent understanding and application of the core concepts (approximately 70%).

Supporting clusters – rethinking and linking; areas where some material is being covered, but in a way that applies core understandings (approximately 20%).

Additional Clusters – expose students to other subjects, though at a distinct, level of depth and intensity (approximately 10%) (New York State Education Department, 2011).

Table 2 is a curriculum calendar from a probability and statistics unit. This plan is based on the old NYS standards which taught types of graphs, mean, median, and mode, and Venn Diagrams all in the seventh grade. In the CCSS these topics have been moved to other grade levels, or other seventh grade topics and more probability standards have been added to require students to do more random sampling and comparisons of populations.

Table 2

Unit Calendar for Seventh Grade Probability and Statistics Unit Aligned to NCTM Standards

Monday	Tuesday	Wednesday	Thursday	Friday
Types of Graphs	Graphs Continued	Frequency Tables	Mean, Median, Mode, Range	Histograms w/ measures of central tendency
Venn Diagrams	Probability of an Event	Probability of Expected outcomes Predicted vs. Experimented	Tree Diagrams	Independent and Dependent Events
Review of Graphs with Jigsaw activity	Review of Probability	Unit Test	Unit Test extended day if needed	

Table 3 was written by Bromley, Jovell, and Sobolewski from Boces Q III, recovered from nyscirs.org. Its purpose is to compare the old NYS Mathematics Standards to the new CCSS. As you will see, there are six new topics under probability and statistics that students will be expected to learn in the seventh grade. The CCSS for probability and statistics in the seventh grade are very different from the NYS standards that were being implemented in previous years. So, you can understand why my previous unit plan will need many revisions. About half of the old content covered will become completely irrelevant in this unit, while other topics will remain the same, many new lessons will also need to be added.

Table 3

CCSS Compared Directly to NCTM Standards for Seventh Grade Mathematics to Identify New Topics

Core vs NYS Standards

Grade 7	Core	NYS
Ratios and Proportional Relationships ----------		**7.RP**
Proportional Relationships		
	Unit rates with fractions	**New Topic**
	Test for proportionality numerically, in tables, or from a graph	**NT**
	Determine the constant of proportionality verbally, visually, graphically, and from equations	**NT**
	Represent proportions by equations	Same
	From a linear graph of proportionality explain the meaning of (x,y), (0,0), and (1, r) if r is the unit rate	**NT**
	Use proportions to solve multistep ratio and percent problems (interest, tax, markup, markdown, tip, commission, % interest)	**NT**
Number System ----------		**7.NS**
Fraction Fluency		
	Maintain addition, subtraction, multiplication, and division of fractions	**NT**
Integers		
	Represent + / - of integers on number line	Same
	Define addition of integers	Same
	Define subtraction of integers as "adding the opposite"	Same
	Define multiplication and division of integers	Same
	Define inverses	Same
	Define rational numbers	Same
	Convert rational numbers to decimals and address terminating and repeating	Same
	Solve problems with rational numbers	Same

Expressions and Equations ---------- 7.EE

Algebraic Expressions		
	Add, subtract, factor, and expand linear expressions with rational coefficient	Same
	Read, write, evaluate, and interpret expressions with variables	Same
	Solve multistep problems with positive and negative rational expressions and equations	**NT**
	Order of operation	Same
	Solve 2 step equations	Multistep
	Solve 2 step inequalities, graph on number line, and interpret results	Positive coefficient 1 step

Geometry ---------- 7.G

Geometric Figures	Scale Drawings	Same
	Construct triangles given angle / side measures to determine existence	**NT**
	Describe cross-sections of right prisms or pyramids	**NT**
Angle Measure, Area, Surface Area, and Volume		
	Derive the relationship between area and circumference	Same
	Know formulas for area and circumference of circles	Same
	Define supplementary, complementary, vertical, and adjacent angle relationships	**NT**
	Write and solve equations to find unknown angle measures of figures	**NT**
	Solve area, volume, and surface area problems	Extended

Statistics and Probability ---------- 7.SP

Sampling		
	Understand the meaning of a random sample	**NT**
	Generate data from multiple random sample	**NT**
	Examine variability between samples	**NT**
Populations		
	Using random sampling, compare populations by using measures of center and spread	**NT**
	Graphically compare populations	**NT**

Probability		
	Define probability, P(E) as $0 \le P(E) \le 1$	Same
	Distinguish between theoretical and empirical probabilities	Same
	Calculate probabilities	Same
	Represent the sample space for compound events numerically, in tables or trees	Same
	Calculate compound probabilities	Same
	Use simulations to determine frequencies for compound events	**NT**

How will they be implemented?

A major difference from old standards is the implementation teachers are guided to use. The CCSS six instructional shifts needed to implement them effectively for mathematics and ELA. The following is a description of the six shifts in mathematics found at Engageny.org. Shift 1 is focus; teachers should use their time wisely to focus on the concepts that are prioritized in the standards. Teachers should focus their time and energy on building the foundational knowledge that will lead to deep conceptual understanding. Shift 2 is coherence; content should be connected through learning across grades so teachers can rely on their students to have a base of knowledge that can be built upon. Shift 3 is fluency; students are expected to develop speed and accuracy with simple calculations, class time will be set aside for memorization and repetition of the necessary skills. This will allow students more ease when delving into more complex topics that require simple skills on top of new ones. Shift 4 is deep understanding; students will express their understanding thought speech, writing, and applications to new situations. Teachers will support students' abilities to access concepts from many different perspectives. Shift 5 is application; students will be required to tap their knowledge and make unprompted decisions to choose an appropriate concept to solve application problems. Shift 6 is dual intensity; this means students are practicing and understanding with intensity. Teachers should ensure that students are given adequate opportunities to make use of their skills through extended application of math concepts, with the most time spent on the most important concepts (New York State Education Department, 2011).

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The new standards provide four course pathways for mathematics, which are meant to be models, not mandates (National Governors Association Center for Best Practices, 2010). The course names and unit design are open to interpretation, with a set of standards to be used as a guideline to follow. The pathways, beginning in high school, include a traditional pathway typically seen in the U.S., an integrated pathway typically seen internationally, and "compacted" versions of both which will begin in seventh grade to allow students to take college level courses during their senior year (National Governors Association Center for Best Practices, 2010). The pathways are ideally similar, critical areas in courses are identical, but are presented in different orders. This will allow schools to organize their courses in a manner that will be optimal for their students to take advantage of the different organizational offerings (National Governors Association Center for Best Practices, 2010). While assessments for these course pathways have not yet been completed, a solid backing for the need of these pathways has been demonstrated.

> "Watered-down courses which leave students uninspired to learn, unable to catch up to their peers and unready for success in postsecondary courses or for entry into many skilled professions upon graduation from high school are neither necessary nor desirable. The results of not providing students the necessary supports they need to succeed in high school are well-documented. Too often, after graduation, such students attempt to continue their education at 2-or 4-year postsecondary institutions only to find they must take remedial courses, spending time and money mastering high school level skills that they should have already acquired. This, in turn, has been documented to indicate a greater chance of these students not meeting their postsecondary goals, whether a certificate program, two- or four-year degree. As a result, in the workplace, many career pathways and advancement may be denied to them. To ensure students graduate fully prepared, those who enter high school underprepared for high school mathematics courses must receive the support they need to get back on course and graduate ready for life after high school" (National Governors Association Center for Best Practices, 2010).

Chapter 3

Unit Plan

Unit Calendar

Monday	Tuesday	Wednesday	Thursday	Friday
Probability of an Event	Expected Outcomes & Tree Diagrams	M&M Activity	Independent and Dependent Events	Comparing Populations using Central Tendency
Review	Unit Test			

Common Core State Standards

Statistics and Probability 7.SP

Use random sampling to draw inferences about a population.

1. Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.
2. Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.

Draw informal comparative inferences about two populations.

3. Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.
4. Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.

Investigate chance processes and develop, use, and evaluate probability models.

5. Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring.
Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.
6. Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.
7. Develop a probability model and use it to find probabilities of events.
Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.
a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.
b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.
For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?
8. Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.
a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.
b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.
c. Design and use a simulation to generate frequencies for compound events. For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?

Lesson Plans

DAY 1: Probability of an Event

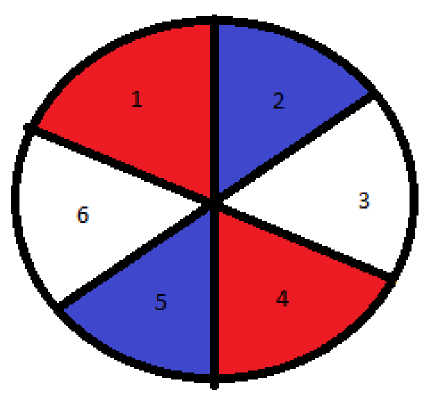
NYCCSS: 7.SP.5, 7.SP.6, 7.SP.7
Goals: The students will be able to understand and calculate probability as a number between 0 and 1.
The students will be able to apply this knowledge to make predictions about events.

Materials: Smart Board, Measuring Tape or Growth Chart, Student Note Sheet, Ticket out the door.

Lesson Procedure: Use the Smart Board to provide students with the following notes (15- 20 minutes). Complete the following activity (12-15 minutes). Class discussion and ticket-out-the-door (10 min or remaining time).

Student Notes:
Probability is a number between 0 and 1.
Probability is written as a ratio, percent, or fraction in lowest terms.
Outcome- possible occurrences, each of which is equally likely to happen.
Sample Space- a list of all the possible outcomes.
Event- any group of similar outcomes.
Probability of an event- ratio of the number of favorable outcomes vs. possible outcomes

$$P(E)=\frac{\#\text{Favorable}}{\#\text{ Possible}}$$



Examples:
1. Spinning a Spinner

  a) P(5)=

  b) P(Red)=

  c) P(7)=

  d) P(not white)=

  e) P(red, blue, or white)=

2. Rolling a Die

a) P(6)=

b) P(even)=

c) P(prime)=

d) P(7)=

e) P(less than 7)=

3. Deck of 52 playing cards

4 suits (Hearts, Diamonds, Spades, Clubs)

a) P(Queen)=

b) P(Black)=

c) P(Face Card)=

d) P(Jack or Red)=

<u>Activity</u>:

Students will collect data in class by measuring the height of each student. A tape measure will be set up on the wall where each student may come up to be measured. One student will keep track of each height in inches on the Smart board as we go. The students will then work independently to solve what percent of the class is above and below them in height. The students will then calculate the probability of the teacher calling on a student that is taller than them at random, and the probability of student that is shorter than them at random. As a class, we will discuss our results.

<u>Ticket-out-the-Door</u>:

1. What is the probability that I will roll a 2 on a six sided die?
2. Using this probability, how many times should a die land on two if I roll it 600 times?
3. Do you think that the die would actually land on 2 that many times if we performed this experiment? Why or why not?

DAY 2: Probability of Expected Outcomes: Theoretical Vs. Experimental and Tree Diagrams

NYCCSS: 7.SP.1, 7.SP.2, 7.SP.5, 7.SP.8
Goals: Students will be able to find compound probabilities using organized lists, data tables, and tree diagrams.
Students will be able to distinguish between theoretical and experimental probability.
Students will understand that probability does not guarantee a certain outcome, but the more experiments that are done will form results closer to the expected outcome.

Materials: Smart Board, Student Note Sheet

Lesson Procedure: Assign warm-up to be completed in 1-2 minutes. Use the Smart Board to provide students with the following notes (30- 35 minutes). Allow students to ask any questions to clarify misunderstandings, and close the lesson (5-10 minutes). Assign homework and allow students to start working (remaining time).

Warm-up Question: If I hold a standard deck of 52 playing cards, what is the probability that you will choose:

a) A red card?
b) A blue card?
c) A Queen or a King?

Student Notes:

There are two ways to calculate probability

1. Use math to predict an outcome
2. Observe an event and keep score

Theoretical probability- uses math to predict an outcome by dividing the number of favorable outcomes by the number of possible outcomes.
Experimental probability- is based on observing a trial or experiment, counting the favorable outcomes, and dividing it by the total number of trials performed. To do this, it is helpful to make a data/frequency table.

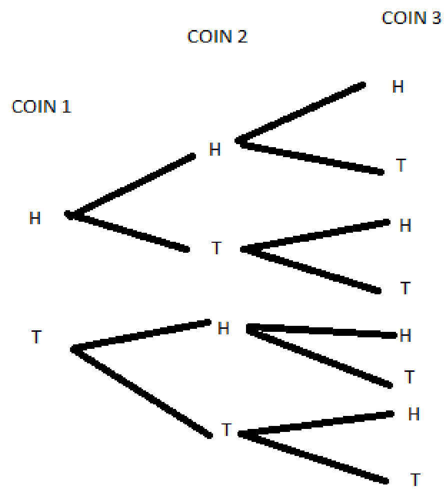
Examples:

1. Miss Redino tossed a quarter 36 times, these are her results.
H, T, H, H, T, T, H, T, T, T, T, T,
T, T, T, H, H, T, H, H, T, H, H, H,
H, T, H, H, T, H, H, T, H, H, T, H

Heads came up 19 times and Tails came up 17 times for a total of 36 flips.
Theoretical probability of flipping heads is 50% and flipping tails is 50%
Experimental probability of flipping heads is 19/36 or 53% and flipping tails is 17/36 or 47%

Tree Diagram- displays a list of all the possible outcomes in sample space in a triangular shape.

Examples:
1. 3 fairs coins are tossed, make a tree diagram and list the sample space.



a) How many outcomes are there total?

b) P(exactly 2 heads)=

c) P(at least 1 tail)=

Fundamental Counting Principle- If there are a ways for one activity to occur, and b ways for a second activity to occur, then there are a • b ways for both to occur.

Examples:
1. Its laundry day, and all of your school clothes are in the washer except for 3 shirts, 4 pants, and two pairs of shoes to choose from. How many different choices for outfits do you have that morning?

3 • 4 • 2 = 24 outcomes

2. You are ordering a pizza. There is a choice of 3 cheeses, 3 crusts, and 12 toppings, how many different pizzas are possible if you may only choose one topping, one cheese and one crust?

Homework Assignment:

Tree Diagrams and FCP (Day 2)

Directions: For each of the following construct a tree diagram and list the sample space to help you solve the problems.

1. Two fair coins are tossed simultaneously.
    a. Draw a tree diagram and list the sample space of all possible pairs of outcomes.
    b. Find P(both coins land on tails)=
    c. Find P(no tails)=
    d. Find P(at least one coin lands on heads)=

2. In a family of two children determine the following probabilities:
    a. P(Both boys)=
    b. P(1 boy, 1 girl)=
    c. P(at least one girl)=
    d. P(all girls)=

3. A fair coin and a six sided die are tossed simultaneously. Draw a tree diagram of the following sample space.

4. When two coins and a six-sided die are tossed simultaneously, what is the number of outcomes in the sample space?

Directions: For each of the following problems use the fundamental counting principle to solve. Show all work!

1. A movie theater sells 5 sizes of popcorn (individual, small, medium, large, and jumbo) with 3 choices of toppings (no butter, butter, extra butter). How many possible ways can a bag of popcorn be purchased?

2. Sam's Club sells girls bicycles. There are three different wheel height options (28, 30, and 32), four different colors (pink, purple, gold, and silver), and three different extras options (none, tassels, glitter). How many possible options of girl's bikes are there?

3. New York State issues license plates consisting of 7 letters and numbers. There are 26 possible letters and the letters may be repeated. There are 10 possible digits and the digits may be repeated. How many possible license plates can be issued with three letters followed by four numbers?

DAY 3: **Probability of Expected Outcomes Activity**

NYCCSS: 7.SP.1, 7.SP.2, 7.SP.5, 7.SP.7
Goals: The students will be able to collect data, and display it in multiple manners to draw conclusions.
The students will be able to draw conclusions from a set of data and use these to find the probability of an event occurring.

Materials: 15-20 bags of M&M's per class, M&M activity sheet

Lesson Procedure:
Review answers to the homework assignment from Day 2 (5 minutes). Students will be completing a partner activity using M&Ms. At the beginning of class, review common misconceptions (probability always provides a guaranteed outcome. 0 and 1 cannot be probable. Probability can be less than 0 or greater than 1) with the students as you assign partners, pass out the activity sheets and M&Ms (2 minutes). Students will work with their partners to follow the directions and complete their activity sheets (30 minutes). Close with a class discussion of results and observations (5 minutes).

Assessment: Student learning will be assessed by grading the M&M worksheet for accuracy.

- Rubric
  - 4/4 Work is shown and is completely correct, directions were followed, and students worked together cooperatively.
  - 3/4 Work is shown, with minimal errors, directions were followed, and students worked together well.
  - 2/4 Some work is shown, more than 3 errors, directions somewhat followed, students had issues working together.
  - 1/4 Incomplete work, no work shown, more than 5 errors, directions not followed, but an attempt at working to solve problems with partner was made.
  - 0/4 No work was done, no effort shown.

M & M's and Probability

You and your partner are going to do some "testing" of probability today. DO NOT EAT any M&M's until the worksheet says you can. If you do, your data will be wrong.

One person will be the M&M selector, and the other person will be the Recorder. Pick who will have each job and record it below.

M&M selector ______________________________

Recorder ______________________________

Here are the steps you are going to take:

1. The M&M selector reaches into the bag without looking and picks out one M&M.
2. The Recorder tallies on the chart which color M&M was picked.
3. The selector puts the M&M back in the bag.
4. Repeat steps 1-3 forty-nine more times. (You will make 50 different picks total.)

The Recorder will record the data below.

M&M Color	**Tally of Times Picked**	**Total # of Times Picked**
Red		
Yellow		
Blue		
Green		
Brown		
Orange		

After the Recorder has tallied the number of picks, add the tally marks and record the total number in the column "Total # of Times Picked".

The M&M selector needs to copy the data for the "Total # of Times Picked" only.

Now you can complete the back of this worksheet together.

M&M's and Probability – *Calculations*

1. Count the total number of M&M's in the bag. Record the number here: ____________

2. Calculate the *actual* results that you obtained for each color. Use the "Actual Results" table to the right. You will need to record the equation for each probability.
   (Hint: For this, use the equation $\text{Probability} = \dfrac{\text{Total \# of Times Picked}}{\text{Total \# of M\&M's}}$)

Actual Results		
Color	$\dfrac{\text{Total \# of Times Picked}}{\text{Total \# of M\&M's}}$	**Probability**
Red		
Yellow		
Blue		
Green		
Brown		
Orange		

3. Count the number of each color of M&M's in the bag. Record your answers in "# of M&M's" column in the "Theoretical Probability" table below.

4. Now we are going to calculate the *theoretical* probability using the equation we learned about in class. Write your calculations in the "Theoretical Probability" table below, and record the probability of pulling each color out of the bag. (NOW you can eat the M&M's ☺)

Theoretical Probability			
Color	**# of M&M's**	$\dfrac{\text{\# of M\&M's}}{\text{Total \# of M\&M's}}$	**Probability**
Red			
Yellow			
Blue			
Green			
Brown			
Orange			

M&M's and Probability – *Reflection*

1. Record the probability in the "Actual Results" table and the probability in the "Theoretical Results" table for each color. Compare the two results. Are they the same? If they are not, the same, are they close?

Color	Actual Results (Probability)	Theoretical Probability
Red		
Yellow		
Blue		
Green		
Brown		
Orange		

2. Should the numbers for the "Actual Results" be close to the "Theoretical Probability"? Why or why not?

3. Explain, as if you were explaining it to a student who was absent, how to calculate the probability of an event.

http://www.fengerhighschool.org/ourpages/auto/2010/4/7/34530710/M_M-probability-activity.doc.

DAY 4: Independent and Dependent Events

NYSCC: 7.SP.8ab
Goals: The students will be able to identify when the probability of an event becomes dependent.
The students will be able to calculate the probability of a dependent event by multiplying fractions.

Materials: Deck of playing cards, Student note sheet, 15 bags of colored bingo chips.

Lesson Procedure: Perform the anticipatory set as follows in an area of the room visible to all students (3 minutes). Provide the following notes to the students using the Smart Board (5-7 minutes). Pass out materials and assign partners for students to complete the following activity (20 minutes). Collect the activity sheets and discuss student results (3 minutes). Assign homework and allow the students to start working (remaining time).

Anticipatory Set: Have the students watch as you pull 1 card at random from a deck of 52 playing cards. Without replacing it, choose a second card. Ask the students, "What is the probability that the first card chosen is a queen and the second card chosen is a jack?"
Analysis: The probability that the first card is a queen is 4 out of 52. Because the first card is not replaced, then the second card is chosen from a deck of 51 cards. So, the probability that the second card is a jack given that the first card is a queen is 4 out of 51.
Conclusion: The outcome of choosing the first card has affected the outcome of choosing the second card, making these events dependent.

Student Notes:

What makes an event independent or dependent?

Two events are Independent if the first event does not affect the outcome of the second
EX: a marble is drawn from a bag and then replaced, a second marble is drawn.

Two events are Dependent if the occurrence of the first event affects the outcome of the second so the probability is changed.
EX: 2 marbles are drawn at random and are not replaced.

This can be found by multiplying the probabilities of both events.

$$P(A \text{ and } B) = P(A) \bullet P(B)$$

Activity: Students will be broken into groups of two and given a bag of colored bingo chips (red, blue, green, yellow). Students will empty the contents of the bag and take tally of how many of each color chip they possess. Students will then calculate the individual probability of drawing each color at random. All students will answer the same questions, as follows.

P(Red)=

P(Blue)=

P(Green)=

P(Yellow)=

Directions: For this section of questions replace the bingo chip you have drawn after you pick it.

1. P(Red then Blue) =
2. P(Green then Yellow)=
3. P(Yellow then Blue)=
4. P(Red then Red)=

Directions: For this section of question DO NOT replace the bingo chip after you have picked it.

5. P(Red then Blue) =
6. P(Green then Yellow)=
7. P(Yellow then Blue)=
8. P(Red then Red)=

Homework Assignment:

Independent and Dependent Events (Day 4)

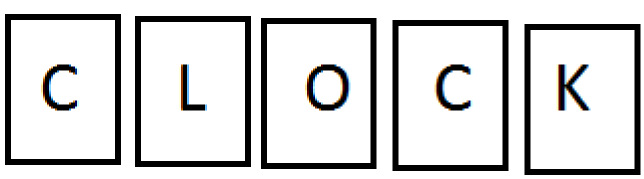
Sometimes, independent events are referred to as events that occur with replacement, and dependent events are events that occur without replacement. This may be helpful to remember when answering problems that involve drawing an item.

Directions: The following questions may be independent or dependent, identify which type the problem is on the line provided, then solve the problem showing all work. All answers should be reduced to lowest terms where necessary.

1. A coin is tossed and 6-sided die is rolled simultaneously. ____________________
    a. P(head on coin)=
    b. P(odd number on die)=
    c. P(heads on coin and odd number on die)=
    d. P(tails on coin and 6 on die)=

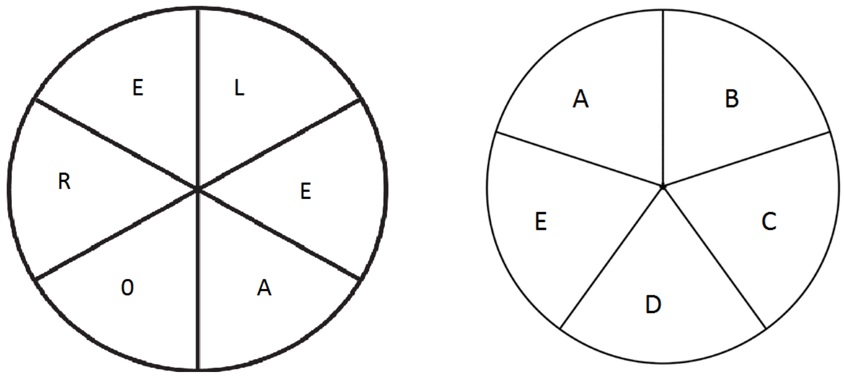
2. There are 16 plants at a florist, 6 of them are tulips. You are going to pick them at random. _______________________
    a. P(buying a tulip)=
    b. P(buying another tulip)=
    c. P(tulip then tulip)=

3. You have a set of flash cards with letters as shown. A card is chosen at random and not replaced. _____________________



    a. P(C)=
    b. P(another C)=
    c. P(C then C)=
    d. P(not K)=
    e. P(not another K)=
    f. P(not K then not K)=

4. Two spinners are spun simultaneously. Find the probability of each outcome. All answers should be reduced to lowest terms. __________________



a. P(both A's)=
b. P(one E and one B)=
c. P(both vowels)=
d. P(one E and one C)=
e. P(both consonants)=
f. P(one R and one C)=

DAY 5: Drawing Informal Comparative Inferences

NYSCC: 7.SP.3, 7.SP.4
Goals: Students will be able to use measures of central tendency (from prior knowledge) and variability to draw conclusions about two populations
Students will be able to calculate mean absolute deviation as a way to draw comparative inferences about two populations.

Materials: Smart Board, Student Note Sheet, Activity Sheet, Computer Lab (Excel)

Lesson Procedure: Review answers from the homework assignment assigned the previous day (5 minutes). Fill in note packet on the Smart Board with the students (10-12 minutes). In the computer lab, have the students use their notes to complete the activity to make inferences on two populations (20-25 minutes).

Student Notes:

Brief Review of Terms:

Measures of Central Tendency

1. Mean – the average of all scores
2. Median – the score corresponding to the 50^{th} percentile
3. Mode – the most frequently occurring score

Measures of Variability

1. Concept of variability – high variability = widely distributed scores around the mean, low variability = most scores lie fairly close to the mean
2. Range – the largest score minus the lowest score
3. Variance – the distance of the number from the mean

Determining Mean Absolute Deviation

1) Determine the Mean: Add all numbers and divide by the count

Ex: the weights of 3 seventh grade students are 82 lbs., 100 lbs., and 127 lbs.

Mean = (82 + 100 + 127)/3

Mean = 103

Determine deviation of each variable from the Mean

Ex: 82-103= -21

100-103= -3

127-103=124

Take the absolute value of the deviations

21, 3, 24

The Mean Absolute Deviation is (21+3+24)/3 =16

This can also be done in an EXCEL file by using the following formula: AVEDEV(82,100,127)

Activity: Students will work independently to complete this activity on the computer. We will be comparing two populations, using the terms we have just gone over in our notes.

1. Heights of the East High School Men's Basketball Team vs. Heights of the East High School Men's soccer team.
   Heights in Inches:
   Basketball: 67, 72, 74, 76, 68, 72, 73, 69, 78, 71, 72, 75
   Soccer: 62, 72, 68, 68, 70, 65, 64, 68, 70, 72, 63, 65

   Find the Mean, Median and Mode for each team: ______ ______ ______
   Which measure of central tendency will be most useful when making comparisons between these two teams? _____________Why? ____
   ____

   Using Excel, Find the Mean Absolute Deviation for each team. This can be calculated in Excel by using the formula AVEDEV( ). List the height of each member separated by a comma in between the parenthesis. Then, organize the data you generate into a table to be used for comparisons.

   Make an informal comparative inference about the two teams based on your calculations
   ____
   ____

2. The following two passages are taken from a 4th grade science book and a 7th grade science book.

4th Grade Science: "Oceans support the greatest variety of life on earth, from microscopic plankton to giant whales. The deepest parts of the oceans have barely begun to be explored, and new life forms are being discovered every year by deep ocean submersible machines. The floor of the ocean is called the benthic habitat while the water itself is called the pelagic habitat. Both support a diversity of animal life. Coral reefs, which grow in warm tropical and subtropical seas, are perhaps the richest marine habitat in terms of the diversity of life they shelter."

7th Grade Science: "The first step towards fossilization for vertebrates, or animals with backbones, is rapid burial in sediment. One scenario might be a drowned animal that washes downstream and lodges into a riverbank, where it quickly becomes covered in sand or mud. The animal's soft parts, namely its flesh and organs, rot away, while sediments surround and protect the animal's hard parts—its bones and teeth. Over time, mineral-rich water percolates through the bone's tiny pores and, gradually, the bones absorb these minerals and turn to stone. But this is only the beginning. For a fossil to survive through time, the surrounding rock must withstand the forces of erosion and tectonic activity as well."

Using an Excel Spreadsheet, design a data table to keep track of the word length in the previous passages. For each passage count and record the word length of a string of 15 consecutive words of your choice. Then, find the mean word length for each passage. Use this to calculate the Mean Absolute Deviation.

Are the words in the seventh grade text book generally longer than the words in the fourth grade text book? _____ Why or why not? Explain using your calculations

____

____

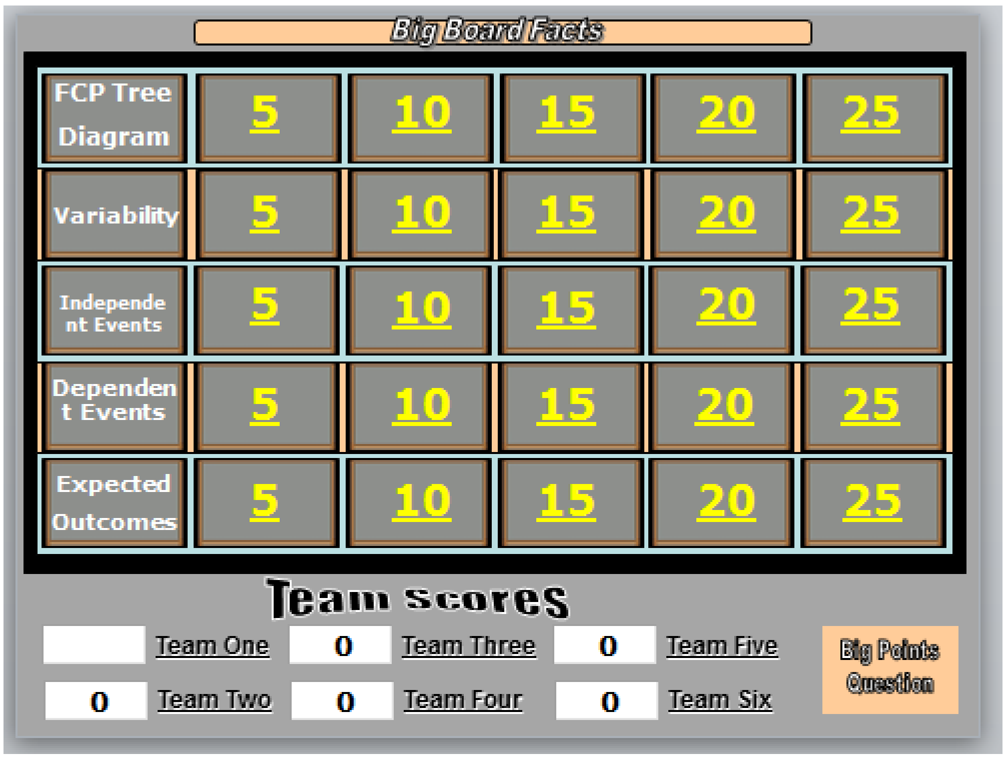
____

DAY 6: Review Game

NYSCC: 7.SP.1-8 (all covered in unit)
Goals: Students will be able to recall information learned from previous lessons and apply learned skills in order to answer questions on probability and statistics.

Materials: Smart Board, mini student white boards or scrap paper.

Lesson Procedure: For review an interactive game of "Big Board Facts" (like Jeopardy) will be played on the Smart Board. Instead of including each slide, just the questions will be listed below. To create this game, the following questions and answers can be typed into a slide show on PowerPoint. Pre-made templates are available at http://people.uncw.edu/ertzbergerj/ppt_games.html#BigBoard. Students will be split into 6 teams, and write their answers to be shown on white boards.



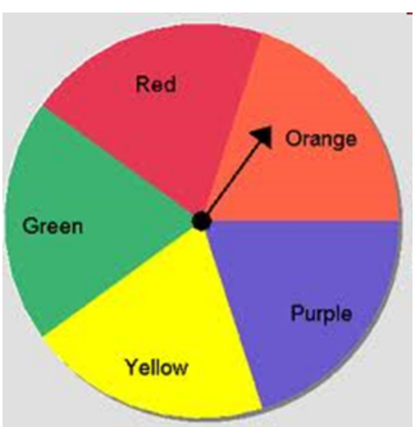
Fundamental Counting Principle/ Tree Diagrams
5. Draw a tree diagram to show all possible outcomes for rolling a die and flipping a coin.
10. How many options do I have to make an outfit with 7 shirts, 4 pants, and 2 pairs of shoes?
15. Draw a tree diagram to show all possible outcomes for flipping 3 coins.
20. How many options do I have to buy a desktop computer with 3 monitors, 5 towers, and 2 keyboards?
25. How many options do I have to buy a car with 12 color options, 2 types of tires, and 2 seat fabrics?

Variability
5. What is the Mode of this set of data: 81, 32, 91, 73, 81, 29, 57, 56?
10. What is the Median of this set of data: 81, 32, 91, 73, 81, 29, 57, 56?
15. What is the Mean of this set of data: 81, 32, 91, 73, 81, 29, 57, 56?
20. Does this set of data have high or low variability?
25. What is the variance of 81?

Independent Events
5. What is the probability that I will flip tails on coin then roll 6 on a die?
10. What is the probability that I will roll anything but a 6, then flip heads on a coin?
15. What is the probability that
20. What is the probability that I will pick a jack then a 9 out of a deck of cards?
25. What is the probability that I will pick the two red queens out of a deck of cards?

Dependent Events
5. A bag of starburst has 7 orange, 5 yellow, 12 pink, and 6 red. What is the total of the sample space?
10. What is the probability of drawing orange then yellow without replacement?
15. What is the probability of drawing pink then red without replacement?
20. What is the probability of drawing orange then orange without replacement?
25. What is the probability of drawing orange or yellow first then pink without replacement?

Expected Outcomes
5. If a coin is tossed 200 times, how many times would you expect to get tails?
10. If you spin this 100 times, how many times would you expect to spin green?



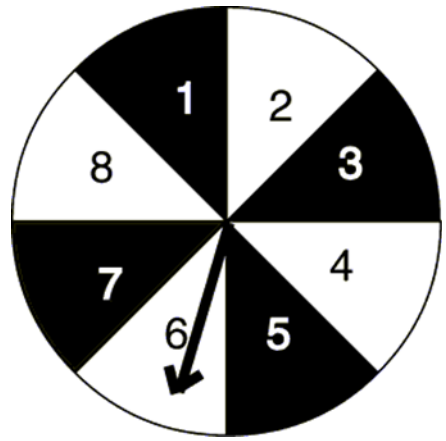
15. Predict how many times out of 360 spins the spinner will land on red or orange.
20. Predict how many times out of 600 rolls that a die will land on an even number.
25. Predict how many times out of 300 rolls that a die will land on 2.

Big Points Question: Describe how to find Mean Absolute Deviation.

DAY 7: Unit Test

SECTION 1: Probability (5pts)

1. Use the Spinner at the right to answer the following questions.



A) P (8) = __________

B) P (even#) = __________

C) P (white) = __________

D) P (9) = __________

E) P (white or black) = __________

SECTION 2: Probability WITH Replacement (SHOW ALL WORK) (6pts)

1. What is the probability that I will flip a coin and get tails, and then roll a 6-sided die and get an even number?

P (Tails, even) =

2. What is the probability that I will flip a coin and get tails, and then roll a 6-sided die and get a 7?

P (Tails, 7) =

3. What is the probability that I will flip a coin and get heads, and then roll a 6-sided die and get anything but 6?

P (Heads, Anything but 6) =

SECTION 3: Probability WITHOUT Replacement (SHOW ALL WORK) (4pts)

1. I am going to pick 3 cards from the pile without replacing them. What is the probability that I will pick the 12, then the 9, then an even number?

P (12, 9, Even) =

SECTION 4: Expected Outcomes (SHOW ALL WORK) (2pts)

1. If you flip a coin 500 times, how many times would you expect to get tails?

2. If you roll a die 330 times, how many times would you expect to get a 2 or a 3?

SECTION 5: Fundamental Counting Principle & Tree Diagrams (4pts)

1. What is the probability that I roll a die and get an odd number and flip a coin and get a heads simultaneously?
    a. Show this with a tree diagram
    b. How many possible outcomes are there for this event? List the sample space and also show using the Fundamental Counting Principle.

SECTION 6: Drawing Inferences & Mean Absolute Deviation (12pts)

1. Find the mean, median, and mode of the following test scores of Student X from marking period 1 and marking period 2.

MP 1: 72, 74, 65, 82 mean: _____ median: ______ mode: ________
MP 2: 98, 80, 91, 89 mean: _____ median: ______ mode: ________

What can you conclude about the student based on the data and your calculations? Use complete sentences ____
____
____

2. The following data shows the heights of three 7th grade math teachers: 68 inches, 72 inches, and 61 inches. Find the mean absolute deviation.

Chapter 4

Reflection

This unit plan was provided to four veteran teachers so that they could critique the lessons using the questionnaire provided in Appendix A. After all of my requests were fulfilled, I received only positive news from everyone. One teacher suggested that his students may not be aware of what comes in a standard deck of cards, because most children do not play cards anymore. I chalked this up to a locational/socio-economic status difference, and chose to keep what I had written in my lesson. It can be altered by user if needed. Other than this, I was told not to change a thing.

I received compliments on the brevity of my unit. This allows for there to be extra days, if needed, to complete a lesson or activity, extend the unit test, or add a project that ties probability and statistics into another unit. Also, the reason I made the unit only 7 days was because probability and statistics is now referred to as an additional topic in the seventh grade curriculum as is shown in Table 1.

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Appendix A

Teacher Questionnaire

1. What do you like about this unit plan and why? Could you see these lessons being taught with success in a seventh grade classroom?

2. What do you dislike about this unit plan? What should be changed and why? In your opinion what aspects of the lessons do you think would be unsuccessful?

3. What suggestions or improvements do you have for me?